Rayat Shikshan Sanstha's

SADGURU GADAGE MAHARAJ COLLEGE, KARAD.

(An Autonomous College - Affiliated to Shivaji University, Kolhapur)

Accredited By NAAC with A+ Grade (CGPA 3.63)

National Education Policy (NEP-2020)

Syllabus for

B.Sc. Part -II

Mathematics

Syllabus to be implemented from July 2023 onwards of

Academic Year 2023-24

Rayat Shikshan Sanstha's

Sadguru Gadage Maharaj College, Karad (Autonomous)

Department of Mathematics

Evaluation Pattern: B.Sc. I Mathematics

(w.e.f. June 2023)

Sem.	Paper Code	Credits	Title of Paper	Evaluation Scheme (Marks)			Grand Total Marks
				CCE	SEE	Total	
I	DSC A5:BMT22-301	02	Real Analysis-I	10	40	50	100
	DSC A6:BMT22-302	02	Algebra-I	10	40	50	
II	DSC B5:BMT22-401	02	Real Algebra-II	10	40	50	200
	DSC B6:BMT22-402	02	Algebra-II	10	40	50	
	DSC A: BMP22-403	04	Lab 1 - Practical-I	100			
	DSC B: BMP22-403		Lab 2 - Practical-I				
Total		12		Total			300

SEE- Semester End Examination**, CCE-** College Compressive Evaluation
Nature of question paper and evaluation scheme:

- Evaluation Scheme
- **Separate passing for Theory, Practical and internal examination is mandatory.**
- In theory examination (**SEE-** Semester End Examination) passing for each paper is at **16** marks (40% of 40 marks).
- In internal of theory examination (**CCE-** Continuous compressive Evaluation) passing for each paper is at **04** marks (40% of 10 marks).
- In practical examination (**SEE-** Semester End Examination) passing is at 40 marks (40% of 100 marks).

SCIENCE STRUCTURE

Levels	Sem.	DSC	DSE/OEC/GEC/IDS	AECC Languages and Env. Sci.	SEC (Multidisciplinary)	Total Credits
	I	4X(4+2)=24 (DSC)	-	1X4=4 (ENG)	SEC-I (1) VBC-I (1)	30
Level-5	II	4X(4+2)=24 (DSC)	-	1X4=4 (ENG)	SEC-II (2)	30
	III	4X(4+2)=24 (DSC)	-	-	SEC-III (2)	26
Level-6	IV	4X(4X2)=24 (DSC)	-	1X4=4 (EVS)	SEC-IV (2)	30
	V	-	4X(2+2)=16 (DSE)	1X4=4 (ENG)	SEC-V (2)	22
Level-7	VI	-	4X(2+2)=16 (DSE)	1X4=4 (ENG)	SEC-VI (2)	22
	VII	-	4X(4+2)=24 (DSE)	-	SEC-VII (2)	26
Level-8	VIII	-	4X(4+2)=24 (DSE)	-	SEC-VIII (2)	26
	Total Credits					212

Rayat Shikshan Sanstha's

Sadguru Gadage Maharaj College, Karad

(An autonomous college)

Department of Mathematics

B.Sc.: 2023-24

Programme Outcome (POs)

1. To acquire the knowledge with facts and figures related to mathematics.
2. To understand the basic concepts, fundamental principles, and the scientific theories related to various mathematical concepts and their relevancies in day-to-day life.
3. To analyze the given scientific data critically and systematically and to draw the objective conclusions.
4. To be able to think creatively (divergently and convergent) to propose novel ideas in explaining facts and figures or providing new solution to the problems.
5. To understand how developments in mathematics helps in the development of other science subjects and vice-versa and how interdisciplinary approach helps in providing better solutions and new ideas for the sustainable developments.
6. To develop scientific outlook not only with respect to mathematics but also in all aspects related to life.

Programme Specific Outcomes (PSOs)

1. To demonstrate basic manipulative skills.
2. To apply the underlying unifying structures of Mathematics and the relationships among them.
3. To demonstrate proficiency in writing proofs.
4. To communicate mathematical ideas both orally and in writing.
5. To investigate and solve unfamiliar mathematical problems.
6. To investigate and apply mathematical problems and solutions in a variety of contexts related to science, technology, business and industry, and illustrate these solutions using symbolic, numeric or graphical methods.

Course Outcomes (COs)

• Real Analysis-I

By the end of the course student will be able to

1) Students will learn to understand basic statements and able to write basic proofs according to principles of quantificational logic
2) Student will understand thoroughly and precisely the concept of “limit” in its various forms
3) Student will understand thoroughly and precisely the concepts of “limit o sequence, bounded sequence, monotonic sequence and Cauchy sequence in its various forms
4) Student will understand sets, equivalent sets, finite, countable and uncountable sets, least upper bound axioms
5) Student will learn to show whether sequence is converges or diverges

• Algebra-I

1) Students should understand types of Matrices and their applications.
2) Students should develop the skills find the Eigen values and Eigen vectors.
3) Present the divisibility and relationship between the Greatest common divisor and least common multiple.
4) Define Types of Matrices, Divisibility in Integers, Equivalence relation and partitions and Congruence relation.
5) Present the concept Group and its basic properties.

• Real Analysis-II

By the end of course, the student will be able to:

1) Ability to work within an axiomatic framework
2) Students will be exposed to the basic ideas of Real Analysis and it is required for their subsequent course work
3) Define Limit Superior and Inferior of Sequences and tests for convergence of series
4) Sequence and series of functions are especially useful in obtaining approximations to a given function and defining new functions from known ones
5) In this paper, we shall consider sequences whose terms are functions rather than real numbers and pay attention to the general properties that are associated with the uniform convergence of sequence and series of functions

• Algebra-II

By the end of course, the student will be able to:

1) Students should understand types of subgroups and how to identify them
2) Students should develop the skills to use various groups and to prove various results
3) Present the relationship between abstract algebraic structures with familiar group theory
4) Define Subgroups, Normal subgroup, Cyclic Subgroups, Homomorphism and Permutation Group
5) Present the concept of Kernel of Homomorphism and Permutation Group structure

Department of Mathematics

Syllabus for B.Sc.-II (Mathematics)

Semester-III, w.e.f. June-2023

Paper- V

BMT22-301 Real Analysis-I (Credits: 02)

Learning Outcomes: Student will have

1. An ability to work within an axiomatic framework
2. A detailed understanding of how monotone sequence converges and completeness property of $\mathbb{R}$ and ability to explain the steps in standard Mathematical notations.
3. Knowledge of some simple techniques for testing the convergence of sequences confidence in applying them;
4. Familiarity with a variety of well-known sequences with a developing intuition about the behavior of new ones;
5. Be able to understand logical arguments and logical constructions. Have a better understanding of sets, functions and relations

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Unit-1: -Sets and Functions

1.1 **Sets**

1.1.1 Operations on sets, Cartesian product of sets, Relation

1.2 **Functions**

1.2.1 **Definitions:** Function, Domain, Co-domain, Range, Graph of a function, Direct image and Inverse image of a subset under a function

1.2.2 **Theorem:** If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$ then

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y)$$

1.2.3 **Theorem:** If $f: A \to B$ and if $X \subseteq B, Y \subseteq B$ then

$$f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y)$$

1.2.4 **Theorem:** If $f: A \to B$and $X \subseteq A , Y \subseteq A$ then $f(X \cup Y) = f(X) \cup f(Y)$

1.2.5 **Theorem:** If $f: A \to B$ and $X \subseteq A , Y \subseteq A$ then $f(X \cap Y) f(X) \cap f(Y)$

1.2.6 **Definitions:** Injective, Surjective and (1-1 correspondence), Bijective function, Inverse function

1.2.7 **Theorem:** Composition of two bijective functions is a bijective function

1.3 **Countable Sets**

1.3.1 **Definitions:** Finite sets, Infinite sets, Countable Sets, Uncountable Sets.

1.3.2 **Examples of Countable sets:** Set of Natural numbers, Set of Integers, Cartesian product of Countable sets

1.3.3 **Theorem:** Countable union of countable set is countable

1.3.4 **Theorem:** Set of Rational numbers is countable

1.3.5 **Theorem:** Any subset of countable set is countable

1.3.6 **Theorem:** The closed interval [0,1] is uncountable

1.3.7 **Theorem:** The set of all real numbers is uncountable

1.3.8 **Theorem:** Every infinite set has a countably infinite subset

1.3.9 **Examples** countability of sets

Unit 2: - Completeness Property of $\mathbb{R}$

2.1 **Definitions:** Lower bound, Upper bound of a subset of $\mathbb{R}$, Bounded set, Supremum $(l.u.b)$, Infimum $(g.l.b)$

2.2 **Least Upper Bound Axiom [Completeness Property of $\mathbb{R}$]**

2.3 **Theorem (Archimedean Property):** If $x \in \mathbb{R}$ then there exists $n_x \in \mathbb{N}$ such that $x \leq n_x$

2.3.1 **Corollary:** If $S = \{\frac{1}{n} : n \in \mathbb{N}\}$ then $\inf S = 0$

2.3.2 **Corollary:** If $t > 0$ then there exist $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t} < t$

2.3.3 **Corollary:** If $y > 0$ then there exist $n_y \in \mathbb{N}$ such that $n_y - 1 < y < n_y$

2.4 **Theorem:** There exists a positive real number x such that $x^2 = 2$

2.4.1 **Corollary:** If x and y are real numbers with $x < y$ then there exist an irrational number z such that $x < z < y$

2.5 **Intervals**

2.5.1 **Characterization Theorem:** If S is a subset of $\mathbb{R}$ that contains at least two points and has the property

If $x, y \in S$ and $x < y$ then the closed interval $[x, y] \subseteq S$ where S is an interval

Unit 3: Sequence of Real Numbers

3.1 **Sequence and Subsequence**

3.1.1 Definition: Sequence, Subsequence and examples.

3.2 **Limit of a Sequence**

3.2.1 **Definition**

3.2.2 **Theorem:** If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non-negative numbers and if $\lim_{n\to\infty} S_n = L$ then $L \geq 0$

3.3 **Convergent Sequence**

3.3.1 **Theorem:** Convergent sequence cannot converge to two distinct points

3.3.2 **Theorem (Without Proof) :** If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then any subsequence of $\{S_n\}_{n=1}^{\infty}$ is also convergent to L

3.4 **Operations on Convergent sequences**

3.4.1 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n + t_n) = L + M$

3.4.2 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n - t_n) = L - M$

3.4.3 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers,
If $c \in \mathbb{R}$ and if $\lim_{n\to\infty} s_n = L$ then $\lim_{n\to\infty} cs_n = cL$

3.4.4 **Theorem:** If $0 < x < 1$ then the sequence $\{x^n\}$ converges to 0

3.4.5 **Lemma:** If sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L then $\{{s_n}^2\}_{n=1}^{\infty}$ converges to L^2

3.4.6 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,
if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$ then $\lim_{n\to\infty}(s_n . t_n) = LM$

3.4.7 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers,
if $\lim_{n\to\infty} s_n = L$ and $\lim_{n\to\infty} t_n = M$, M ≠ 0 then

$$\lim_{n\to\infty}(s_n / t_n) = L/M$$

Unit 4: Monotone Sequences and Cauchy Sequences

4.1 **Monotone Sequence**

4.1.1 **Definition and Examples**

4.1.2 **Theorem:** A non-decreasing sequence which is bounded above is convergent

4.1.3 **Theorem:** A non-increasing sequence which is bounded below is convergent

4.1.4 **Corollary:** The sequence $\{(1 + \frac{1}{n})^n\}$ is convergent

4.1.5 **Theorem (Without Proof):** A non-decreasing sequence which is not bounded above diverges to infinity

4.1.6 **Theorem (Without Proof):** A non-increasing sequence which is not bounded below diverges to infinity

4.1.7 **Theorem:** A bounded sequence of real numbers has convergent subsequence

4.2 **Cauchy Sequence**

4.2.1 **Definition and Examples**

4.2.2 **Theorem:** If sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then $\{s_n\}_{n=1}^{\infty}$ is Cauchy sequence

4.2.3 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is bounded

4.2.4 **Theorem :** If $\{s_n\}_{n=1}^{\infty}$is the Cauchy sequence of real numbers then $\{s_n\}_{n=1}^{\infty}$is Convergent

Recommended Books:

1. **R.R. Goldberg**, Methods of real Analysis, Oxford & IBH Publishing co. Pvt. Ltd, New Delhi (UNIT 1,2,3,4)
2. **S.C. Malik and Savita Arora**, Mathematical Analysis (Fifth Edition), New Age International (P) Limited, 2017(UNIT 1,2,3,4)
3. **T. M. Apostol**, Calculus (Vol.-I), John Wiley and sons (Asia) P.Ltd.2002

Reference Books :

1. **R.G. Bartle and D.R. Sherbert**, Introduction to Real Analysis, Wiley India Pvt. Ltd, Fourth Edition, 2016
2. **D. Somasundaram and B Choudhary**, First Course in Mathematical Analysis, Narosa publishing house New, Delhi, Eighth Reprint 2013.
3. **P.K. Jain and S.K. Kaushik,** An Introduction to Real Analysis, S.Chand & Company Ltd. New Delhi, First Edition 2000
4. **Shanti Narayan and M.D. Raisinghania,** Elements of Real Analysis, S.Chand& Company Ltd. New Delhi, Fifteenth Revised Edition 2014

Paper- VI

Algebra-I (BMT22-102) (Credits: 02)

Learning Outcomes:

1. After successful completion of Matrices students will solve the system of equations using the language of Matrices
2. Understand the elementary concepts of Matrices, System of a linear Equations, Greatest common divisor and least common multiple, Partial order relation and basic structure of Group
3. Elementary number theory is the study of the basic structure and properties of integers
4. Learning Matrices and Divisibility of integers helps improving one's ability of Mathematical Thinking

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Unit 1: Matrices [12]

1.1 **Introduction**

- 1.1.1 Definition with Illustration.
- 1.1.2 Types of Matrices
- 1.1.3 Definitions: Transpose of Matrix, conjugate of Matrix, Symmetric Matrix, Asymmetric Matrix

1.2 **Hermitian and Skew Hermitian**

- 1.2.1 **Definitions: Hermitian** and Skew Hermitian
- 1.2.3 **Theorem:** The necessary and sufficient condition for a matrix A to be Hermitian is that $A = A^{\theta}$.
- 1.2.4 **Theorem:** The necessary and sufficient condition for a matrix A to be Skew Hermitian is that $A^{\theta} = -A$.
- 1.2.5 **Theorem:** If A and B are Hermitian (Skew Hermitian) then A+B is also Hermitian (Skew-Hermitian).
- 1.2.6 **Theorem:** If A is Hermitian then iA is Skew Hermitian.
- 1.2.7 **Theorem:** If A is Skew Hermitian then iA is Hermition.
- 1.2.8 **Theorem:** Every square Matrix is uniquely expressed as the sum of Hermitian Skew Hermitian matrix.

1.3 **Eigenvalues and Eigenvectors**

- 1.3.1 **Definitions:** Minor of matrix, Rank of a matrix, Inverse of a matrix, Characteristics Polynomial of matrix
- 1.3.2 **Eigenvalues and Eigenvectors**

1.3.3 Examples on 1.3.2.

1.4 **System of a linear Equations**

1.4.1 System of Homogeneous linear Equations.

1.4.2 Nature of solutions of $AX = 0$.

1.4.3 Examples on 1.4.1.

1.4.4 System of Non -homogeneous linear Equations.

1.4.5 Nature of Solution $AX = B$.

1.4.6 Examples on 1.4.4.

1.5 **Cayley-Hamilton Theorem (Statement only)**

1.5.1 **Application of Cayley-Hamilton Theorem**.

Unit 2: - Divisibility in Integers [10]

2.1 **Definition: Divisibility in integers**

2.2 **The well ordering principle (Statement Only)**

2.3 **Properties of Divisibility**

2.3.1 Definition of divisor and Multiple

2.3.2 **Theorem:** Let a,b,c,d be integers. Then

i) If a | b then a | bx

ii) If a | b and a | c then a | bx+cy $\forall$ x, y $\in$ I

iii) If a | b and b | c then a | c

iv) If m $\neq$ 0 is in $\mathbb{Z}$ and a | b $\Rightarrow$ am | bm

v) If a | b and c | d then ac | bd

vi) If ab | bc then a | c; (b$\neq$ 0)

2.4 **Theorem: Division Algorithm (Without Proof)**

2.5 **Greatest common divisor and least common multiple**

2.5.1 **Definitions:** Greatest common divisor and least common multiple

2.5.2 **Theorem:** Let a and b be two integers at least one of them not 0. Then there exist

A unique greatest common divisor d of a and b. Moreover, d can be written as $d = am + bn$ for integers m and n.

2.5.3 **The Euclidean Algorithm and Examples.**

2.5.4 Definition: Relatively Prime

2.5.5 **Euclid's lemma:** For a prime number p , if p $\mid$ ab then either p $\mid$ a or p $\mid$ b.

2.6 **Theorem: (Unique Factorization Theorem or Fundamentals Theorem of Arithmetic)**

Unit 3: -Relation [12]

3.1 **Relation**

3.1.1 **Definitions:** Cartesian Product, Relation, Binary Relation, Inverse Relation.

3.1.2 Examples on 3.1.1.

3.2 **Pictorial Representation of Relation**

3.2.1 Co-ordinate Diagram

3.2.2 Arrow Diagram

3.2.3 Matrix Representation

3.2.4 Directed Graph

3.2.5 Examples on 3.2.1 to 3.2.4

3.3 **Composition of Relations**

3.3.1 **Definition:** Composition of Relations

3.3.2 **Theorem:** Let A, B, C and D be sets. Let R: A$\longrightarrow$ B, S: B$\longrightarrow$ C, T: C$\longrightarrow$ D be Relation then $R \circ (S \circ T) = (R \circ S) \circ T$.

3.4 **Types Of Relations**

3.4.1 **Definitions:** Reflexive, Symmetric, Antisymmetric and transitive.

3.4.2 Examples on 3.4.1

3.4.3 **Theorem:** Let R be relation on set A. Then R^{∞} is the smallest transitive relations on A that contain R.

3.5 **Equivalence relation and partitions**

3.5.1 **Theorem:** Let R be an equivalence relation on setA. Then quotient set A/R forms a partition of A .

3.5.2 **Theorem:** Let $\{A_i\}, i \in I$ be partition of a set A. Then there exists an equivalence Relation Ron the set A such that quotient set A / R is the given partition $\{A_i\}, i \in I$ on A.

3.6 **Partial order relation**.

3.6.1 **Definition**: Partial order relation.

3.6.2 Examples on 3.6.1

3.7 **Congruence relation on Integers**

3.7.1 **Definition**: Congruence relation

3.7.2 **Congruence arithmetic**

3.7.3 **Theorem:** Let $n > 1$ be a fixed positive integer and a, b, c, d be arbitrary integers then the following conditions holds

i)If $a \equiv b(\mathrm{mod}\,n)$ and $c \equiv d(\mathrm{mod}\,n)$then $a + c \equiv b + d(\mathrm{mod}\,n)$.

ii)If $a \equiv b(\mathrm{mod}\,n)$ and $c \equiv d(\mathrm{mod}\,n)$ then $ac \equiv bd(\mathrm{mod}\,n)$.

3.7.4 **Examples** on 3.7.3

Unit 4: Groups [11]

4.1 **Binary operation on a set**

4.1.1 **Definition: Binary** operation on a set with illustration

4.2 **Semigroup**

4.2.1 **Definition: Semigroup** with illustration

4.3 **Monoid**

4.3.1 **Definition: Monoid** with illustration

4.4 **Group**

4.4.1 **Definition:** Group, Abelian Group, Finite Group, Infinite Group, Order of a group

4.4.2 Examples on 4.4.1

4.5 **Properties of Groups**

4.5.1 **Theorem:** If $< G,* >$is a group, then

a) Identity element in G is unique

b) Every a in G has unique inverse in G.

c)For every a in G, $(a^{-1})^{-1} = a$

d) For all $a, b \in G$, $(a * b)^{-1} = b^{-1} * a^{-1}$

4.5.2 **Theorem: If** a, b, c are elements in a group G, then

i) $a * b = a * c$ implies $b = c$ (Left Cancellation Law)

ii)$b * a = c * a$ implies $b = c$ (Right Cancellation Law)

4.5.3 **Theorem:** If G is a group and $a,b\in G$, then the equations $a * x = b$ and $y * a = b$ have unique solutions $x = a^{-1} * b$ and $y = b^{-1} * a$ respectively

4.5.4 **Definition:** Order of element with illustration, Properties (Without Proof)

4.6 **Permutations**

4.6.1 **Definition with Illustration**

4.6.2 **Cyclic Permutation**

4.6.3 **Transposition, Disjoint Permutations, Even and Odd Permutations**

Recommended Books:

1. **Shantinarayan**, A Text Book of Matrices, S. chand Co.,Pvt. Ltd. Raminagar, New Delhi. (Unit 1)
2. **David. M. Burton,** Elementarary Number Theory ,7th Edition.2017 McGraw Hill Education
 (Unit 2)
2. **Schaum'sOutline**, Discrete Mathematics,(3rd Edition) : Seymour hipschutz,Marehipson , Tata MaGraw-Hill Publishing Company Ltd., New Delhi.(Unit)
3. **V.K. Khanna and S. K. Bhambri** , A course in abstract Algebra , Vikas Publishing house Private Limited ,New Delhi , Fifth Edition 2016 (Unit 4)

Reference Books :

1. **J.B.Fraleigh**, A first couse in abstract Algebra ,Narosa Publishing House New Delhi, Tenth Reprint 2003
2. **A.R. Vasishtha** ,Modern Algebra ,Krishna Prakashan ,Meerut 1994
3. **M.Artin** ,Algebra , Prentice hall of India ,New delhi,1994
4. **I.N. Herstein** ,Topics in Algebra ,Wiley India Pvt. Ltd.

Paper-VII

Real Analysis-II (BMT22-401) (Credits: 02)

Learning Outcomes: Student will have:

1. An ability to work within an axiomatic framework
2. A detailed understanding of how Cauchy's Criterion for the convergence of sequence and series follow from the completeness property of $\mathbb{R}$ and ability to explain the steps in standard Mathematical notations.
3. Knowledge of some simple techniques for testing the convergence of sequences and series and confidence in applying them;
4. Familiarity with a variety of well-known sequences and series, with a developing intuition about the behavior of new ones;
5. An understanding of how elementary functions can be defined by power series, with an ability to deduce some of their easiest

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Unit 1: Limit Superior and Inferior of Sequences **[14]**

1.1 **Definitions and Examples**

1.1.1 **Theorem:** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then

$$\lim_{n\to\infty} sup\, s_n = \lim_{n\to\infty} S_n$$

1.1.2 **Theorem:** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers then

$$\lim_{n\to\infty} inf\, S_n = \lim_{n\to\infty} S_n$$

1.1.3 **Theorem:** If $\{S_n\}_{n=1}^{\infty}$ is convergent sequence of real numbers and If $\lim_{n\to\infty} sup\, S_n = \lim_{n\to\infty} inf\, S_n = L$ Where $L \in \mathbb{R}$ then $\{S_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n = L$

1.1.4 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers andif $s_n \le t_n$ then

$i)\ \lim_{n\to\infty} sup\, s_n \le \lim_{n\to\infty} sup\, t_n$ $\qquad$ $ii)\ \lim_{n\to\infty} inf\, s_n \le \lim_{n\to\infty} inf\, t_n$

1.1.5 **Theorem:** If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then

$$i)\ \lim_{n\to\infty} sup\,(s_n + t_n) \le \lim_{n\to\infty} sup\, s_n + \lim_{n\to\infty} sup\, t_n$$
$$ii)\ \lim_{n\to\infty} inf\,(s_n + t_n) \ge \lim_{n\to\infty} inf\, s_n + \lim_{n\to\infty} inf\, t_n$$

Unit 2: Series of Real Numbers **[09]**

2.1 **Convergent and Divergent Series**

2.1.1 **Definitions:** Convergent Series, Divergent Series and Examples

2.1.2 If $\sum_{n=1}^{\infty} a_n$ is convergent series then $\lim_{n\to\infty} a_n = 0$

2.2 **Cauchy's General Principal for convergence (Statement only)**

A necessary and sufficient condition for the convergence of an infinite series $\sum_{n=1}^{\infty} u_n$ is that the sequence of its partial sum $\{s_n\}$ is convergent

2.3 **Series of Non-negative real numbers**

2.3.1 **Definition and Examples**

2.3.2 **Theorem:** A positive term series converges if and only if its sequence of partial sum is bounded above

2.4 **Tests for convergence**

2.4.1 **Comparison Test (First Type)**

If $\sum u_n$ and $\sum v_n$ are two positive term series and $k \neq 0$, a fixed positive real number (independent of n) and there exists a positive integer m such that $u_n \leq kv_n$ for every $n \geq m$ then

$a)$ $\sum u_n$ is convergent if $\sum v_n$ is convergent and

$b)$ $\sum v_n$ is divergent if $\sum u_n$ is divergent

2.4.2 **Comparison Test (Second Type)**

If $\sum u_n$ and $\sum v_n$ are two positive term series and there exist positive number m , such that $(u_n/u_{n+1}) \geq (v_n/v_{n+1})$ for every $n \geq m$ then

$a)$ $\sum u_n$ is convergent if $\sum v_n$ is convergent and

$b)$ $\sum v_n$ is divergent if $\sum u_n$ is divergent

2.4.3 $\boldsymbol{p}$ **−Series Test:** The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$

2.4.4 **Root Test:** Consider the series $\sum_{n=1}^{\infty} a_n$ .Then

$a)$ If $\lim sup|a_n|^{1/n} < 1$ then the series convergent absolutely

$b)$ If $\lim sup|a_n|^{1/n} > 1$ then the series diverges

$c)$ If $\lim sup|a_n|^{1/n} = 1$,this test gives no information

2.5 **Alternating Series**

2.5.1 **Leibnitz Test:** If the alternating series

$u_1 - u_2 + u_3 - u_4 + \ ... \ , (u_n > 0$ for every $n)$ is such that

$i)$ $u_{n+1} \leq u_n$, for every n and

$ii)$ $\lim u_n = 0$ then the series converges

2.6 **Examples** on convergence of series

2.7 **Absolute and Conditional Convergence**

2.6.1 Definition and Examples

2.6.2 Theorem: Every Absolutely convergent series is convergent

2.8 **Examples** on absolute convergence of series

Unit 3: Sequence and Series of Functions

3.1 **Pointwise convergence of sequence of functions**

3.1.1 Definition and Examples

3.2 **Uniform convergence of sequence of functions**

3.2.1 Definition and Examples

3.3 **Uniform Convergence and Continuity**

3.3.1 **Theorem:** Assume $f_n \to f$ uniformly on an interval S.If each function f_n is continuous at a point p in S then the limit function f is also continuous at p

3.3.2 **Theorem:** If series of functions $\sum u_k$ converges uniformly to a function f on a set S and if each term u_k is continuous at a point p in S then f is also continuous at p

Unit 4: Differentiability and Integrability of Series of functions

4.1 **Theorem:** Assume that$f_n \to f$ uniformly on an interval $[a, b]$ and assume that each function f_n is continuous on $[a, b]$.Define a new sequence $\{g_i\}$ by the equation $g_n(x) = \int_a^x f_n(t)\ dt$ if $x \in [a, b]$ and $g(x) = \int_a^x f(t)\, dt$ Then $g_n \to n$ uniformly on $[a, b]$ .In symbols, we have

$$\lim_{n\to\infty} \int_a^x f_n(t)\, dt\ =\ \int_a^x \lim_{n\to\infty} f_n(t)\, dt$$

4.2 **Theorem:** Assume that series of functions $\sum u_k$ converges uniformly to a function $f$on an interval $[a, b]$ where each u_k is continuous on $[a, b]$.
For $x \in [a, b]$
define$g_n(x) = \sum_{k=1}^n \int_a^x u_k(t)\ dt$ and $g(x) = \int_a^x f(t)\ dt$
Then $g_n \to g$ uniformly on $[a, b]$

4.3 **Sufficient Condition for Uniform Convergence**

4.3.1 **Theorem (Weierstrass M-Test):** Given series of functions $\sum u_k$ which converges
pointwise to a function $f$on a set S.If there is a convergent series of positive constants $\sum M_n$ such that $0 \le |u_n(x)| \le M_n$ for every $n \ge 1$ and every x in S. Then $\sum u_k$converges uniformly on$S$.

4.4 **Power Series**

4.4.1 **Definition**

4.4.2 **Interval of Convergence and its examples**

Recommended Books:

1. **R.R.Goldberg**,Methods of real Analysis, Oxford & IBH Publishing co. Pvt. Ltd, New Delhi (UNIT 1,2,3,4)
2. **S.C.Malik and SavitaArora**, Mathematical Analysis(Fifth Edition), New Age International (P) Limited, 2017(UNIT 1,2,3,4)
3. **Tom M Apostol**,Calculus (Vol.I) ,John Wiley and sons (Asia) P.Ltd.2002

Reference Books:

1. **R.G.Bartle and D.R.Sherbert**, Introduction to Real Analysis, Wiley India Pvt. Ltd, Fourth Edition, 2016
2. **D. Somasundaram and B Choudhary**, First Course in Mathematical Analysis, Narosa publishing house New, Delhi, and Eighth Reprint 2013.
3. **P.K.Jain and S.K.Kaushik**, An Introduction to Real Analysis, S.Chand & Company Ltd. New Delhi, First Edition 2000
4. **Shanti Narayan and M.D.Raisinghania**, Elements of Real Analysis, S.Chand & Company Ltd. New Delhi, Fifteenth Revised Edition 2014
5. **Shanti Narayan andP.K.Mittal**, A course of Mathematical Analysis, S.Chand & Company Ltd. New Delhi, Reprint 2016

Paper- IV

BMT22-402 Algebra-II (Credits: 02)

Learning Objectives: students will be able to: -

1. Understand the basic concepts of Subgroups and their applications in both algebraic and geometric structures
2. Explain the fundamental concepts of algebra and their role in modern algebra
3. Explain Demonstrate accurate and efficient use of advanced algebraic techniques
4. Apply problem-solving using advanced algebraic techniques applied to diverse situations in Mathematics

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Unit 1: - Subgroups **[12]**

1.1 **Subgroups**

1.1.1 **Definition:** Subgroups with illustrations

1.2 1.2.1 **Theorem:** A non-empty subset H of a group G is a subgroup of G If and only if

(i) $a, b \in H \Rightarrow ab \in H$

(ii) $a \in H \Rightarrow a^{-1} \in H$

1.2.2 **Theorem:** A non-empty subset of a group G is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$

1.2.3 **Theorem:** A non-empty finite subset H of a group G is a subgroup of G iff H is Closed under multiplication.

1.3 **Centre of a Group**

1.3.1 **Definition:** Centre of a Group, Normalizer of a element with illustration.

1.3.2 **Theorem:** Centre of group G is subgroup of group G.

1.3.3 **Theorem:** Normalizer of an element group G is subgroup of group G.

1.4 **Cosets**

1.4.1 **Definition:** Coset and examples

1.4.2 **Theorem:** Let H be a subgroup of G then

i) $Ha = H \iff a \in H \; and \; aH = H \iff a \in H$

ii) $Ha = Hb \iff ab^{-1} \in H$ and $aH = bH \iff a^{-1}b \in H$

iii) $Ha(aH)$ is a subgroup of G iff $a \in H$

1.3.3 **Theorem:** $Ha = \{x \in G \mid x \equiv a \; mod \; H\} = cl(a)$ for any a in G

1.4 **Lagrange's Theorem**

1.4.1 Theorem: If G is a finite group and H is a subgroup of G then $o(H)$ divides

$o(G)$.

1.5 **Index of a subgroup**

1.5.1 **Definition:** Index of subgroup H in G with illustration

1.6 **Theorem: For** subgroups H and K of G, HK is a subgroup of G iff HK=KH

Unit-2: - Cyclic Groups [11]

2.1 **Cyclic groups**

2.1.1 **Definition:** Cyclic group, generator of a cyclic group

2.1.2 Examples on 2.1.1

2.2 2.2.1 **Theorem:** Order of a cyclic group is equal to the order of its generator.

2.2.2 **Theorem:** A subgroup of cyclic group is cyclic.

2.2.3 **Theorem:** Every cyclic group is abelian.

2.2.4 **Theorem:** If G is finite group, then order of any element of G divides order of G.

2.2.5 **Theorem:** An infinite cyclic group has precisely two generators.

2.3 **Euler ϕ function**

2.3.1 **Definition:** Euler's $\emptyset$ function

2.3.2 **Theorem:** Number of generators of a finite cyclic group of order n is $\emptyset(n)$.

2.4 **Euler and Fermat's Theorem**

2.4.1 **Euler's Theorem:** Let $a, n(n \geq 1)$ be any integers such that $\gcd(a, n) = 1$ then $a^{\emptyset(n)} \equiv 1(\mathrm{mod}\,n)$

2.4.2 **Fermat's Theorem:** For any integer a and prime p $a^{p} \equiv a(\mathrm{mod}\,p)$

2.4.3 Examples on 2.4.1 and 2.4.3

Unit-3: - Normal groups [11]

3.1 **Normal groups**

3.1.1 Definitions: Normal subgroups, Simple group

3.1.2 Examples

3.2 **Results on Normal Groups**

3.2.1 **Theorem:** A subgroup H of group G is normal in G iff$g^{-1}Hg = H, g \in G$.

3.2.2 **Theorem:** A subgroup H of group G is normal in G iff $g^{-1}hg \in H$ for all $h \in H, g \in G$.

3.2.3 **Theorem:** A subgroup H of group G is normal in G iff the product of two right (left) cosets of H in G is again a right (left) coset of H in G.

3.3 **Quotient groups**

3.3.1 **Definition: Quotient groups with illustration.**

3.3.2 **Theorem:** If G is finite group and N is normal subgroup of G then $o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}$.

3.3.2 **Theorem:** Every quotient group of cyclic group is cyclic.

Unit-4: - Homomorphism, Permutation Group [11]

4.1 Homomorphism

4.1.1 Definitions: Homomorphism, Epimorphism, Monomorphism, Endomorphism and Automorphism.

4.1.2 Examples on 4.4.1

4.1.3 **Theorem:** If $f: G \to G'$ is homomorphism then

(i) $f(e) = e'$

(ii) $f(x^{-1}) = [f(x)]^{-1}$

(iii) $f(x^n) = [f(x)]^n$, nis an integer.

4.2 Kernel of Homomorphism

4.2.1 **Definition:** Kernel of Homomorphism with illustration

4.2.2 **Theorem:** If $f: G \to G'$ is homomorphism then kerf is a normal subgroup of G.

4.2.3 **Theorem:** A homomorphism $f: G \to G'$ is one-one if and only if $\ker f = \{e\}$.

4.3 Isomorphism Theorems

4.3.1 **Fundamental Theorem of group Homomorphism:** If $f: G \to G'$ is an onto homomorphism with $K = \ker f$ then $\frac{G}{K} \cong G'$.

4.3.2 **Second theorem of Isomorphism :(Statement only)** Let H and K be two subgroups of group G, where H is normal in G, then $\frac{HK}{H} \cong \frac{K}{H \cap K}$.

4.3.3 **Third theorem of Isomorphism (Statement only):** Let H and K be two normal subgroups of group G, such that $H \subseteq K$ then $\frac{G}{K} \cong \frac{G/H}{K/H}$.

4.4 Permutation Group

4.4.1 **Cayley Theorem:** Every group G is isomorphic to a permutation group.

4.4.2 **Theorem:**Set of even permutations is a normal subgroup of S_n Alternating group.

Recommended Books:

1. **V.K. Khanna and S. K. Bhambri**, A course in abstract Algebra, Vikas Publishing house Private Limited ,New Delhi , 3rd Edition 2008 (Unit 1,2,3,4)

Reference Books:

1. **J.B.Fraleigh**, A first course in Abstract Algebra, Narosa Publishing House New Delhi,
   Tenth Reprint 2003.
2. **A.R. Vasishtha**, Modern Algebra, Krishna Prakashan, Meerut 1994
3. **M.Artin**, Algebra , Prentice hall of India ,New delhi,1994
4. **I.N. Herstein**, Topics in Algebra ,Wiley India Pvt. Ltd.

BMP22-403 Mathematical Practical-I

Sr. No.	Name of the practical	No. of Practical
1	Solution of System of m linear homogeneous equations in n unknowns	1
2	Solution of System of m linear non homogeneous equations in n unknown	1
3	Inverse of Matrix by Cayley Hamilton Method	1
4	Euclidean Algorithm	1
5	Pictorial Representation of Relation	1
6	Examples on equivalence relation	1
7	Examples on Fermat’s theorem	1
8	Examples on Group & Order of an element	1
9	Beta function	1
10	Gamma function	1
11	Examples on Cyclic Group	1
12	Examples on Normal Subgroup	1
13	Permutation Group	1
14	Homomorphism and Group	1
15	Comparison test and Cauchy’s Root test	1
16	D’Alembert’s Ratio test and P-test	1
17	Double Integration over the given region	1
18	Double Integration: Change of order of integration	1
19	Double Integration: Change of co-ordinate axis	1
20	Double Integration by using Polar Co-ordinates	1

BMP22-403 Mathematical Practical-II

Sr. No.	Name of the practical	No. of Practical
1	C-Introduction-I	1
2	C-Introduction-II	1
3	Complete Structure of C-program	1
4	Simple C-program	1
5	If Statement, If else Statement & Switch Statement	1
6	While loop & do while loop	1
7	For loop	1
8	Go to, break continue statement	1
9	One Dimensional Array	1
10	Two-Dimensional Array	1
11	Function	1
12	Trapezoidal Rule and its Program	1
13	Simpsons (1/3)rd rule and program	1
14	Simpsons (3/8)th rule and program	1
15	Gauss Elimination Method	1
16	Gauss Jordan Method	1
17	Gauss-Seidel Method	1
18	Euler's Method	1
19	Euler's Modified Method	1
20	Runge-Kutta second & fourth order Method	1

Department of Mathematics

Nature of SEE Question Papers

(w.e.f. June 2023)

Que. 1. Select correct alternative. **08**

1. ....

a) .... b) ....

c) .... d) ....

2. ....

a) .... b) ....

c) .... d) ....

3. ....

a) .... a) ....

c) .... c) ....

4. ....

a) .... a) ....

c) .... c) ....

5. ....

a) .... a) ....

c) .... c) ....

6. ....

a) .... b) ....

c) .... d) ....

7. ....

a) .... b) ....

c) .... d) ....

8. ....

a) .... b) ....

c) .... d) ....

Que. 2. Attempt any two. **16**

A) ....

B) ....

C) ....

Que. 3. Attempt any four. **16**

a) ....

b) ....

c) ....

d) ....

e) ....

f) ....